

Noise-induced looping on the Bloch sphere: Oscillatory effects in dephasing of qubits subject to broad-spectrum noise

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For many implementations of quantum computing, $1/f$ and other types of broad-spectrum noise are an important source of decoherence. An important step forward would be the ability to back out the characteristics of this noise from qubit measurements and to see if it leads to new physical effects. For certain types of qubits, the working point of the qubit can be varied. Using a new mathematical method that is suited to treat all working points, we present theoretical results that show how this degree of freedom can be used to extract noise parameters and to predict a new effect: noise-induced looping on the Bloch sphere. We analyze data on superconducting qubits to show that they are very near the parameter regime where this looping should be observed.

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Motivated by the prospect of quantum computation and communication, coherent quantum operation and control of small systems has become a central area of physics research. The isolation of these systems from external noise is a key problem, as noise produces decoherence. In solid-state systems, some level of $1/f$ or other broad-spectrum noise (BSN) is almost always present, and is typically difficult to eliminate [1]. Indeed, in superconducting qubits, single-electron and other tunneling devices, this type of noise is recognized as the factor chiefly responsible for dephasing [2, 3, 4, 5].

One interesting question is the extent to which the characteristics of the BSN can be determined by measurements on the qubit itself. This has been considered by several authors [6, 7, 8, 9]. For the most part, these authors considered the case of pure dephasing noise. Some theoretical work has been done for "mixed" noise, which causes both relaxation and dephasing, but this has usually been limited to Gaussian approximations [10], asymptotic analysis, or small numbers of RTNs [11].

In this Letter, we show how to treat mixed noise analytically for all times, fully taking into account the non-Gaussian effects. This will enable us to show how to back out the characteristics of the noise from qubit measurements. A new physical effect is predicted: noise-induced looping on the Bloch sphere. We shall analyze data on superconducting flux qubits to show that they are close to the regime in which this effect comes into play. However, we stress that this effect can occur in any two-level system that is subject to BSN, which can include qubit implementation from atomic and molecular physics as well as solid-state ones.

The effective Hamiltonian of a qubit is often written as $H = -\frac{1}{2}(\varepsilon\sigma'_z + \Delta\sigma'_x) - \frac{1}{2}\vec{h}'(t) \cdot \vec{\sigma}'$, where ε and Δ are the energy difference and tunneling splitting between the two physical states. For example, in a flux qubit ε is proportional to the applied flux through the superconducting loop and Δ is the Josephson coupling. $\vec{h}'(t)$ is the noise, a random function. We shall be interested in

the case where $\vec{h}'(t)$ comes from K random telegraph noise sources (RTNs) with a wide range of switching frequencies, giving rise to BSN. The coordinate system will be rotated by the angle $\theta = \tan^{-1}(\Delta/\varepsilon)$ so that $\sigma'_z = \cos\theta\sigma_z - \sin\theta\sigma_x$ and the qubit energy eigenstates are along the z -axis. θ is called the working point of the qubit and H is then

$$H = -\frac{1}{2}B_0\sigma_z - \frac{1}{2}\sum_{k=1}^K s_k(t) \vec{g}_k \cdot \vec{\sigma} \quad (1)$$

where $B_0 = \sqrt{\varepsilon^2 + \Delta^2}$. \vec{g}_k is the coupling of the k -th RTN to the qubit. $s_k(t)$ switches randomly between the values -1 and 1 with an average switching rate γ_k . Together, these sources generate the random field $\vec{h}(t) = \sum_{k=1}^K s_k(t) \vec{g}_k$. We assume throughout that $B_0 \gg h(t)$. \vec{g}_k is at an angle θ_k to the z -axis, which can be thought of as the angle between the noise axis and the energy axis. The pure dephasing case, the basis of much work in this field, corresponds to $\theta_k = 0$. We deal only with classical noise, which is generally thought to be appropriate for the systems of interest here. Eq.1 is general enough to describe almost any qubit subject to classical noise, since an arbitrary power spectrum can be constructed with appropriate choices of coupling constants \vec{g}_k and rates γ_k . For simplicity, we drop throughout any azimuthal dependence of the noise and $\vec{g}_k = g_k(\sin\theta_k, 0, \cos\theta_k)$.

In many cases, the RTN sources have a fixed direction. For example, it has been shown that for superconducting flux qubits the chief noise source is flux noise [12], which would put the noise along the z -axis (ε direction), and gives $\theta_k = \theta$. We also stress that the working point θ is tunable for many different qubit architectures, either by changing ε or Δ [13, 14, 15].

A convenient solution method for noise from an ensemble of RTNs with an arbitrary distribution of \vec{g}_k and γ_k has recently been given [16]. This "quasi-Hamiltonian" method lends itself to perturbative treatment for both fast (weak-coupling) RTNs: $g_k \cos\theta_k \ll \gamma_k$ and slow

(strong-coupling) RTNs: $g_k \cos \theta_k \gg \gamma_k$. The occurrence of the trigonometric factor in these inequalities is crucial: it comes from non-analyticity at the point $g_k \cos \theta_k = \gamma_k$ which leads to qualitatively different behavior in the two regimes. The method can treat experiments that involve an arbitrary sequence of control pulses. Because it gives analytic results for the non-Gaussian effects of BSN over the whole range of θ , we can identify new systematic effects.

In this work we shall restrict attention to two common experimental protocols: Energy Relaxation (ER), which measures $n_{ER}(t) = \langle \sigma_z(t) \rangle$, and Spin Echo (SE), which measures $n_{SE}(t) = \langle \sigma_x(t) \rangle$ with a π -pulse at $t/2$. Additional information about the noise may be available by the use of more complicated pulse sequences [8].

The solutions so obtained are:

$$n_{ER}(t) \simeq \exp \left[\left(-2 \sum_{m=1}^M \gamma_m \epsilon_{2m}^2 \sin^2 \theta_m + \Gamma_1 \right) t \right] \quad (2)$$

$$n_{SE}(t) \simeq e^{-(\Gamma_2 + \Gamma_3)t} \left[1 + \sum_{m=1}^M \epsilon_{1m} \sin(g_m t \cos \theta_m) \right] \quad (3)$$

where M is the total number of slow RTNs and N is the total number of fast RTNs, labeled by m and n respectively; $M + N = K$. Γ_1 and Γ_2 are the energy relaxation and dephasing rates caused by fast RTNs, $\Gamma_3 = \sum_m \gamma_m$ is the overall dephasing rate from slow RTNs. $\epsilon_{1m} = \gamma_m / (g_m \cos \theta_m)$ and $\epsilon_{2m} = g_m / B_0$ are the small parameters of the perturbation theory. For any value of θ , the portion of the spectrum where the perturbation theory breaks down is a small fraction of the whole, so we expect the approximation to be well-controlled.

The slow RTNs' contribution to the ER signal is much smaller than Γ_1 and thus can be neglected. RTN is non-Gaussian noise, which accounts for the relatively complicated appearance of the formulas. We wish to use observations of these signals to back out information on the distribution of the \vec{g} 's and γ 's.

The fast RTNs' contribution to qubit decoherence is fully captured by two decay constants Γ_1 and Γ_2 . For them, our results coincide with Redfield theory [17] and

$$\Gamma_1 = \sum_{n=1}^N \frac{2\gamma_n g_n^2 \sin^2 \theta_n}{4\gamma_n^2 + B_0^2}, \quad (4)$$

$$\Gamma_2 = \frac{\Gamma_1}{2} + \Gamma_\phi, \quad \text{where } \Gamma_\phi = \sum_{n=1}^N \frac{g_n^2 \cos^2 \theta_n}{2\gamma_n}. \quad (5)$$

These fast RTNs give rise to purely exponential decay, as is well known [18].

In contrast, the slow RTNs give rise to the Γ_3 term in the exponent of n_{SE} , which gives exponential decay, but also to the more complex and θ_m -dependent factors. These factors give rise to decays which are quasi-

Gaussian over short range of time and possibly oscillations at longer times.

For purposes of fitting, it is necessary for us to specify a not completely general but yet still flexible model for the noise. Let $n(\gamma) = \sum_{k=1}^K \delta(\gamma - \gamma_k)$ be the distribution of rates and take constant values $g_k = g$, $\theta_k = \theta$. If there is a range of couplings then g in the following formulas can be regarded as the root-mean-square coupling. We will assume a broad noise spectrum by taking $n(\gamma) = \alpha \gamma^{s-1}$ for $\gamma_{\min} < \gamma < \gamma_{\max}$ and 0 otherwise. Note $s = 0$ gives $1/f$ noise. This power-law assumption is often useful for analyzing experimental data while the method itself is capable of treating arbitrary distributions. The task of data analysis is then to determine g , α , s , γ_{\min} , and γ_{\max} from observations of $n_{SE}(\theta, t)$. The data are relatively insensitive to γ_{\max} , making it difficult to determine. We will comment on other limitations of the model below. The results of the fitting can be used to make predictions for future experiments.

With these assumptions, the previous results can be simplified into

$$n_{ER}(t) \simeq e^{-\Gamma_1 t}, \quad (6)$$

$$n_{SE}(t) \simeq e^{-(\Gamma_2 + \Gamma_3)t} \left[1 + \frac{\Gamma_3}{\gamma_c} \sin(\gamma_c t) \right], \quad (7)$$

where $\gamma_c = g \cos \theta$ is the critical coupling strength that separates fast and slow RTNs and $\Gamma_3 = \int_{\gamma_{\min}}^{\gamma_c} n(\gamma) \gamma d\gamma$. The key point is that by tuning the working point θ , one effectively changes the relative number of fast and slow RTNs in the environment. Since the two kinds of RTNs have different qualitative effects on the qubit, we can use this fact to get information about the noise.

Given these results, it is convenient to define

$$\begin{aligned} \Phi_{SE}(t, \theta) &= \frac{n_{SE}(\theta)}{n_{SE}(\theta = \pi/2)} \\ &= e^{-\Gamma_3 t} \left[1 + \frac{\Gamma_3}{\gamma_c} \sin \gamma_c t \right], \end{aligned} \quad (8)$$

where $\Phi_{SE}(t, \theta = 0)$ correspond to the "phase-memory functional" defined by other authors [11]. Note $\Gamma_2(\theta) \simeq \Gamma_1(\theta = \pi/2)/2$, thus it drops out in Eq.8.

We shall now use these formulas to determine some parameters of the noise experienced by the flux qubit (sample A) in Ref.[14]. For this system, $\Delta/h = 5.445$ GHz and ε/h varies from 0 to 1 GHz, meaning that $\cos \theta$ varies from 0 to about 0.2. Fig.1 shows the fit of Eq.8 to the experimental results at $\cos \theta = 0.18$. This gives $\Gamma_3 = 0.99$ MHz and $\gamma_c = 2.1$ MHz. The fit is certainly excellent, but we note that a Gaussian, as used by the authors of Ref.[14] fits equally well over this time range, with only a hint of deviation at the longest times. The working point can be calculated from $\cos \theta = \varepsilon/B_0$, thus fitting $\gamma_c(\theta)$ determines $g/h = 9.6$ MHz, as shown in Fig.2(a). Fitting the data of $\Gamma_3(\theta)$ in Fig.2(b) gives $s = 0$

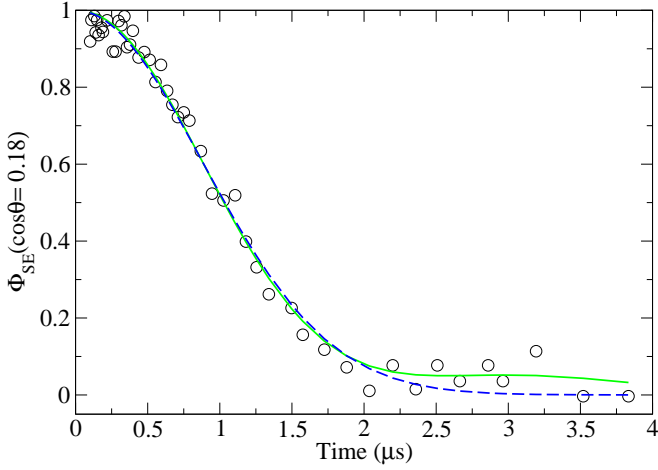


FIG. 1: (Color Online) Echo phase memory functional Φ_{SE} data in Fig.4A of Ref.[14]. We fit the 36 data points to Eq.8 (green solid line) and Gaussian model $\Phi_{SE}^G = e^{-\Gamma_G^2 t^2}$ (blue dashed line) respectively. The center of the first plateau is thus predicted to be at $\tau_P = 5\pi/2\gamma_c \simeq 3.7\mu s$.

(true 1/f noise), $\alpha = 0.77$ and $\gamma_{\min} = 0.048$ MHz. Note this dependence is very sensitive to the value of s thus can be used to identify the noise distribution $n(\gamma)$. We have carried out the analysis for the results of a similar experiment of Ref.[15]. All results are given in Table I. An interesting consistency check is to compute Γ_1 from the model with the parameters determined since Γ_1 can be fitted from $n_{ER}(t)$ independently. For the data of Ref.[14] (column A of Table I), we find $\Gamma_1^{(th)} = 0.02$ MHz, while the measured value is $\Gamma_1^{(ex)} = 0.65$ MHz. This strongly suggests an additional source of high-frequency noise not captured by our noise model. For the data of Ref.[15] (column B), we find $\Gamma_1^{(th)} = 5.4$ MHz and $\Gamma_1^{(ex)} = 5.7$ MHz; this shows that the noise model is reasonably complete for this experiment.

The experiments are typically fit by the Gaussian theory for dephasing noise [10, 19]. Galperin *et al.* [11] have pointed out the importance of non-Gaussian effects and have shown numerically that these effects appear in $\Phi_{SE}(t)$ at strong coupling and longer times. This takes

	A	B
Δ/h (GHz)	5.445	3.9
ε/h (GHz)	0 ~ 1	0 ~ 0.925
$\Gamma_1^{(ex)}$ (MHz)	0.65	5.7
g/h (MHz)	9.6	135
γ_{\min} (MHz)	0.048	0.098
$n(\gamma)$	$0.77/\gamma$	$0.75/\gamma$
$\Gamma_1^{(th)}$ (MHz)	0.02	5.4

TABLE I: Noise characteristics extracted from Ref.[14] (column A) and Ref.[15] (column B).

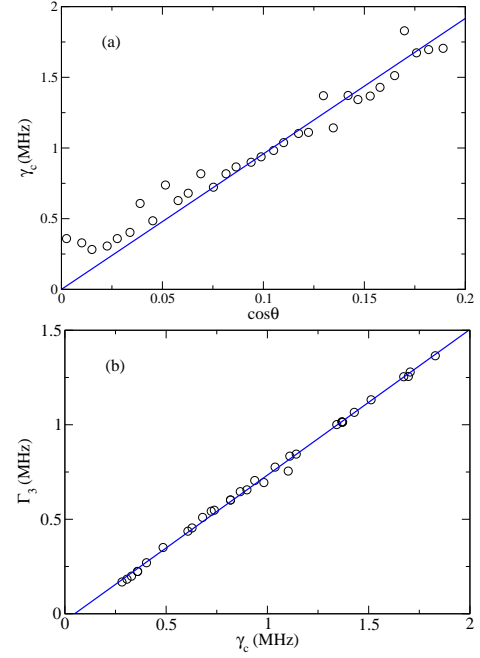


FIG. 2: Fitting of sample A data from Ref.[14]. Both γ_c and Γ_3 are fitted from Φ_{SE} at various working point. (a) Critical rate γ_c versus the working point $\cos\theta$. (b) Linear regression to $\Gamma_3 = \alpha(\gamma_c - \gamma_{\min})$.

the form of "plateaus". Eqs.7 and 8 allow us to quantify and systematically analyze these deviations from Gaussian behavior at arbitrary θ .

Note that for BSN, α is the parameter that controls the appearance of the plateaus for 1/f noise. Physically, it describes the overall scale of the noise since $K = \int_{\gamma_{\min}}^{\gamma_{\max}} n(\gamma) d\gamma$. Fig.3 shows the behavior of $\Phi_{SE}(t)$ for various values of α . In these plots we have used the parameters of Ref.[14] and varied only α . The experimental value $\alpha = 0.77$ is closest to Fig.3(b). Fig.3(c) shows that the experiments are not far from seeing some non-Gaussian behavior.

Fig.3 shows that as α is reduced a series of plateaus becomes visible and the phenomenon is better described as oscillations, as would be expected from Eq.8. This could be done by reducing the number of RTNs, which is presently a very active area of research [20]. Fig.3(d) shows these plateaus quite clearly, and confirms that the period of these oscillations is given by $2\pi/(g \cos\theta)$. The observation of a period that is adjustable by changing the working point would be an unambiguous signature of the effect.

The physical interpretation of the oscillations is simple. $\Phi_{SE}(t)$ is a measurement of the probability that the Bloch vector has returned to its starting point on the equator in the rotating frame. The Gaussian theory treats the sphere as a plane and the decoherence as a random walk on that plane. In the more rigorous theory, there is an additional process - a return to the starting

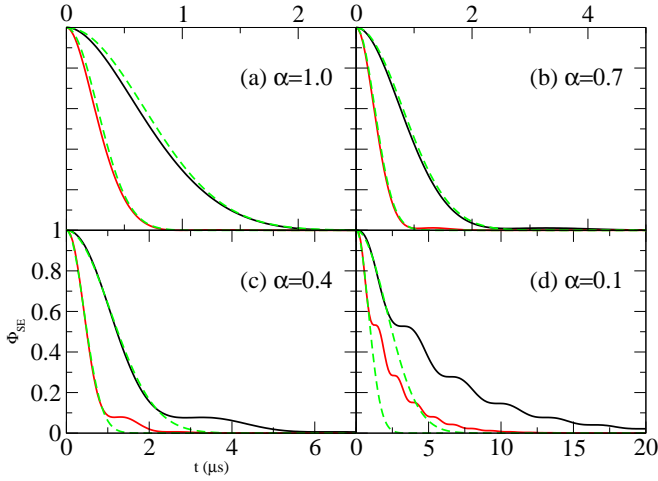


FIG. 3: (Color Online) Φ_{SE} with different noise intensity α . In all subfigures, solid black line corresponds to $\cos \theta = 0.2$ and solid red line corresponds to $\cos \theta = 0.5$. The corresponding Φ_{SE} calculated from Gaussian approximation are plotted as dashed green lines. Note $\Gamma_G \propto \sqrt{\alpha} \cos \theta$ and Gaussian approximation works well in the short time region until the emergence of the first plateau, if there is one. All subfigures share the same axes labels as in (c).

point after looping the sphere. This process is only possible for the slow RTNs whose fields can persist over a time long enough for the loop to occur. It is due to motion along lines of latitude on the sphere and only the z-component of these fields produces this. This explains why the characteristic looping time is $2\pi/(g \cos \theta)$.

This looping depends on having not too much spread in the distribution of the \tilde{g}_k . Clearly a very wide spread would wash out the oscillations, as seen in Eq.3. On the other hand, a wide spread that weights the frequency components of the power spectrum unequally is not consistent with the usual picture of the origin of $1/f$ noise as being due essentially to a spread in γ_k .

In conclusion, we have exploited a new mathematical method to obtain the qubit decoherence behavior as a function of working point θ for qubits that are subject to BSN coming from classical sources. Varying θ changes the ratio of strongly and weakly-coupled RTNs. It therefore furnishes a convenient method to back out noise parameters from observations of the qubits themselves. This allows us to refine the picture of non-Gaussian oscillations in qubit decoherence. In particular we have supplied a method to observe the oscillations, and have identified their physical origin as noise-induced looping on the Bloch sphere. This looping should be observable in many different qubit implementations.

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